SCIENCE CHINA
Mathematics

• ARTICLES •

March 2015 Vol. 58 No. 3: 525–530
doi: 10.1007/s11425-014-4960-6

Plurigenera of compact connected strongly pseudoconvex CR manifolds

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Received August 15, 2013; accepted December 5, 2014; published online December 18, 2014

Abstract Strongly pseudoconvex CR manifolds are boundaries of Stein varieties with isolated normal singularities. We introduce a series of new invariant plurigenera $\delta_m, m \in \mathbb{Z}_+$ for a strongly pseudoconvex CR manifold. The main purpose of this paper is to present the following result: Let X_1 and X_2 be two compact strongly pseudoconvex embeddable CR manifolds of dimension $2n-1 \geqslant 3$. If there is a non-constant CR morphism from X_1 to X_2, then $\delta_m(X_2) \leqslant \delta_m(X_1)$ where $\delta_m(X_i)$ is the plurigeneus of X_i (see Definition 2.4).

Keywords plurigenera, strongly pseudoconvex, CR manifold

MSC(2010) 32V15, 14B05

Citation: Lin K P, Yau S, Zuo H Q. Plurigenera of compact connected strongly pseudoconvex CR manifolds. Sci China Math, 2015, 58: 525–530, doi: 10.1007/s11425-014-4960-6

1 Introduction

Intuitively, one can think of CR manifold as boundary of complex manifold. Abstractly it can be defined as follows.

Definition 1.1. Let X be a compact connected orientable manifold of real dimension $2n-1$, $n \geqslant 2$. A CR structure on X is a rank $n-1$ subbundle S of the complexified tangent bundle $\mathbb{C}T(X)$ such that

(1) $S \cap \overline{S} = \{0\}$

(2) If L, L' are local sections of S, then so is $[L, L']$.

The manifold X, together with the CR structure S, is called a CR manifold. There is a unique subbundle $\mathcal{H}$ of $T(X)$ such that $\mathbb{C}\mathcal{H} = S \oplus \overline{S}$. Furhtermore, there is a unique homomorphism $J : \mathcal{H} \to \mathcal{H}$ such that $J^2 = -1$ and $S = \{v - iJv : v \in \mathcal{H}\}$. The pair $(\mathcal{H}, J)$ is called the real expression of the CR structure.

Definition 1.2. Let X_1 and X_2 be connected CR manifolds of dimension $2n-1$. X_1 is said to be a wiggle of X_2 if $X_1 \cup X_2$ bounds a complex manifold of n-dimensional.

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Definition 1.3. Let $L_1, \dots, l_{n-1}$ be a local frame of the CR structure S on X so that $\overline{L}_1, \dots, \overline{L}_{n-1}$ is a local frame of $\overline{S}$. Since $S \oplus \overline{S}$ has complex codimension one in $\mathbb{C}T(X)$, we may choose a local section N of $\mathbb{C}T(X)$ such that $L_1, \dots, L_{n-1}$, $\overline{L}_1, \dots, \overline{L}_{n-1}$, N span $\mathbb{C}T(X)$. We may assume that N is purely imaginary. Then the matrix (c_{ij}) defined by

$$[L_i, \overline{L}_j] = \sum_k a_{ij}^k L_k + \sum_k b_{ij}^k \overline{L}_k + \sqrt{-1} c_{ij} N$$

is Hermitian, and is called the Levi form of X.

The Levi form is noninvariant; however, its essential features are invariant. For example, let (c_{ij}) have eigenvalues $\lambda_1, \dots, \lambda_{n-1}$, take the signature of (c_{ij}) to be the number of positive eigenvalues minus the number of negative eigenvalues, then the number of non-zero eigenvalues and the absolute value of the signature of (c_{ij}) are independent of the choice of $L_1, \dots, L_{n-1}, N$.

Definition 1.4. X is said to be strongly pseudoconvex if the Levi form is definite.

Theorem 1.5 (See [2]). *Let X be a compact strongly pseudoconvex CR manifold of real dimension $2n-1 \geqslant 5$. Then X is CR embeddable in some $\mathbb{C}^N$.*

On the other hand, Rossi et al. [10] showed that there exists a compact 3-dimensional strongly pseudoconvex CR manifold not embeddable in $\mathbb{C}^N$. In this paper, we assume that the compact CR manifold X of real dimension $2n-1$ is already embeddable in $\mathbb{C}^N$.

A C^1 function $f : X \to \mathbb{C}$ is a CR holomorphic function if it satisfies the tangential Cauchy Riemann equations $Yf = 0$ for all $Y \in S$. $\phi : X_1 \to X_2$ is said to be a CR biholomorphic map from CR manifold X_1 to CR manifold X_2 if ϕ is a diffeomorphism and ϕ and ϕ^{-1} are CR holomorphic map. Two CR manifolds are called CR equivalent if there exists a CR biholomorphic map between them.

A central problem of CR geometry asks: Given two strongly pseudoconvex CR manifolds X_1 and X_2, how can one distinguish them?

The following theorem of Harvey and Lawson [4] plays a fundamental role in this central problem.

Theorem 1.6 (See [4]). *Let X be a compact connected strongly pseudoconvex embeddable CR manifold. Then there exists a unique complex variety V in $\mathbb{C}^N$ for some N such that the boundary $\partial V = X$ and V has only normal isolated singularities.*

For any compact connected strongly pseudoconvex embeddable CR manifold X, we introduce a notion of plurigenera $\delta_m(X), m \geqslant 1$, which is a non-negative integer and is invariant under CR biholomorphism. $\delta_m(X)$ measures the complexity of the CR manifold. In this paper, the main results are as follows.

Main theorem. *Let X_1 and X_2 be two compact connected, $(2n-1)$-dimensional $(2n-1 \geqslant 3)$, strongly pseudoconvex embeddable CR manifolds. If there is a non-constant CR morphism from X_1 to X_2, then $\delta_m(X_1) \geqslant \delta_m(X_2)$.*

An immediate corollary of the main theorem is as follows.

Corollary. *Let X_1 and X_2 be two compact connected, $(2n-1)$-dimensional $(2n-1 \geqslant 3)$, strongly pseudoconvex embeddable CR manifolds. If there is a positive integer m such that $\delta_m(X_1) < \delta_m(X_2)$, then there is no non-constant CR morphism from X_1 to X_2.*

In Section 2, we shall introduce a series of new invariant plurigenera on strongly pseudoconvex varieties. These new invariants can be used to measure the complexity of the CR manifold. In Section 3, we shall give the proof of our main theorem.

2 Plurigenera of compact connected strongly pseudoconvex CR manifolds

In view of an example of Webster [13], it is clear that the problem of studying when two given CR manifolds are analytically equivalent is very difficult. Webster example suggests that it is difficult to study the wiggles of a CR structure (see Definition 1.2). Luk and Yau [6, 7] have introduced a notion of algebraic equivalence relation among CR manifolds. If a CR manifold is a wiggle of another CR manifold, then they are algebraically equivalent. In some sense, in order to understand the strata of the moduli space of embeddable CR structures which are not a wiggle of each other, we need to study algebraic equivalence among embeddable CR structures.

Definition 2.1. Let X_1 and X_2 be two compact connected, $(2n-1)$-dimensional, strongly pseudoconvex embeddable CR manifolds which bound complex varieties V_1, V_2 of dimension n, respectively in $\mathbb{C}^N$. Let $\tilde{V}_1$ and $\tilde{V}_2$ be the normalization of V_1, V_2, respectively. X_1 is said to be algebraically equivalent to X_2 if the corresponding normal varieties $\tilde{V}_1$ and $\tilde{V}_2$, which are bounded by X_1 and X_2, respectively, have isomorphic singularities $\tilde{y}_1$ and $\tilde{y}_2$, i.e., $(\tilde{V}_1, \tilde{y}_1) \cong (\tilde{V}_2, \tilde{y}_2)$ as germs of varieties.

It was observed in [8] that two CR equivalent manifolds are automatically algebraically equivalent.

Proposition 2.2 (See [8])**.** *Let X_1 and X_2 be two connected compact strongly pseudoconvex CR manifolds in $\mathbb{C}^N$. If X_1 is CR equivalent to X_2, then X_1 is algebraiclly equivalent to X_2 .*

Luk et al. [8] also introduced some numerical invariants under algebraic equivalence for connected compact strongly pseudoconvex embeddable CR manifolds of real 3-dimensional. In particular, the geometric genus $p_g(X)$ of the CR threefold X was introduced. In this paper, we introduce a series of new invariants $\delta_m(X)$ for any strongly pseudoconvex CR manifold X of dimension $2n-1$. $\delta_m(X)$ measures the complexity of the CR manifold.

Let X be a connected compact strongly pseudoconvex CR manifold of real dimension $2n-1$ and $n \geqslant 2$. Suppose that X bounds a normal variety $V \subset \mathbb{C}^N$ with isolated singularities $Y = \{q_1, \ldots, q_m\}$. Let $\pi : (M, A) \to (V, Y)$ be a resolution of singularities $Y \subset V$ with the exceptional set A. Let V' be any sufficiently small Stein neighborhood of Y, $M' := \pi^{-1}(V')$, and $U := V' \setminus Y$, i.e., we have $\pi : (M', A) \to (V', Y)$. Let K be the canonical line bundle of U. Let U_j be an open covering of U with coordinates $(z_j^1, \ldots, z_j^n)$. For any positive integer m, a section $w \in \Gamma(U, \mathcal{O}(mK))$ is regarded as an m-ple holomorphic n-form and written on U_j as

$$w = \phi_j(z_j)(dz_j^1 \wedge \cdots \wedge dz_j^n)^{\otimes m}.$$

We associate with w the continuous (n,n)-form $(w \wedge \bar{w})^{1/m}$, given on U_j by

$$(w \wedge \bar{w})^{1/m} = |\phi_j(z_j)|^{\frac{2}{m}} \left(\frac{\sqrt{-1}}{2}\right)^n dz_j^1 \wedge d\bar{z}_j^1 \wedge \cdots \wedge dz_j^n \wedge d\bar{z}_j^n.$$

Definition 2.3. An m-ple holomorphic n-form $w \in \Gamma(U, \mathcal{O}(mK)$ is said to be $L^{2/m}$-integrable if

$$\int_{W \setminus Y} (w \wedge \bar{w})^{1/m} < \infty,$$

for any sufficiently small relatively compact neighborhood W of $Y \subset V$. We denote by $L^{2/m}(U)$ the set of $L^{2/m}$-integrable m-ple holomorphic n-forms on U, which is a subspace of $\Gamma(U, \mathcal{O}(mK))$. $L^{2/m}(U)$ becomes a vector space $\Gamma(M', \mathcal{O}(mK + (m-1)A))$ by Sakai [11, Theorem 2.1, p. 243]. As for $L^{2/m}(M' - A)$ we replace V' and Y with M' and A, respectively in the definition of $L^{2/m}(U)$. It is easy to see that $L^{2/m}(U) = L^{2/m}(M' - A)$.

Definition 2.4. Let X be a connected compact strongly pseudoconvex CR manifold of real dimension $2n-1$ and $n \geqslant 2$. The plurigenus (m-genus), m a positive integer, of X is defined by $\delta_m(X) := \dim_{\mathbb{C}} \Gamma(U, \mathcal{O}(mK))/L^{2/m}(U)$.

We will show that $\delta_m(X)$ is finite and independent of the choice of the Stein neighborhood of Y.

Let us first recall the following useful lemma.

Lemma 2.5 (See [5])**.** *Let $\pi : M \to V$ exhibit A as exceptional in M with V a Stein space. If $M \supset N$, with N a holomorphically convex neighborhood of A and $\mathcal{F}$ is a coherent analytic sheaf on M, then the restriction map $\rho : H^i(M, \mathcal{F}) \to H^i(N, \mathcal{F})$ is an isomorphism for $i \geqslant 1$.*

With the same notion as before, i.e., let V' be any sufficiently small Stein neighborhood of Y, $M' := \pi^{-1}(V')$, and $U := V' \setminus Y$, we have $\pi : (M', A) \to (V', Y)$. Following Laufer [5], we consider the sheaf cohomology with support at infinity. The following sequence is exact:

$$0 \to \Gamma(M', \mathcal{O}(mK)) \to \Gamma_\infty(M', \mathcal{O}(mK)) \to H^1_c(M', \mathcal{O}(mK)) \to \cdots$$

By Siu [12, p. 374], any section of mK defined near the boundary of M' has an analytic continuation to $M' - A$. Therefore, there is a natural isomorphism

$$\Gamma_\infty(M', \mathcal{O}(mK)) \cong \Gamma(M' - A, \mathcal{O}(mK)).$$

By Serre duality,

$$H^1_c(M', \mathcal{O}(mK)) \cong H^{n-1}(M', \mathcal{O}(K - mK)).$$

Since M' is strongly pseudoconvex, $H^{n-1}(M', \mathcal{O}(K-mK))$ is finite dimensional. Hence, by the inequality

$$\begin{aligned}\dim\Gamma(M' - A, \mathcal{O}(mK))/L^{2/m}(M' - A) &\leqslant \dim\Gamma(M' - A, \mathcal{O}(mK))/\Gamma(M', \mathcal{O}(mK))\\ &\leqslant \dim H^1_c(M', \mathcal{O}(mK)) = \dim H^{n-1}(M', \mathcal{O}(K - mK)),\end{aligned}$$

we have $\dim\Gamma(M' - A, \mathcal{O}(mK))/L^{2/m}(M' - A) < +\infty$. If $V' \supset V''$, with V'' another Stein neighborhood of Y, $M' := \pi^{-1}(V')$ and $M'' := \pi^{-1}(V'')$, then we have the following exact sequences:

$$\begin{aligned}0 &\to \Gamma(M', \mathcal{O}(mK)) \to \Gamma(M' - A, \mathcal{O}(mK)) \to H^1_c(M', \mathcal{O}(mK))\\ &\to H^1(M', \mathcal{O}(mK)) \to \cdots\end{aligned}$$

and

$$\begin{aligned}0 &\to \Gamma(M'', \mathcal{O}(mK)) \to \Gamma(M'' - A, \mathcal{O}(mK)) \to H^1_c(M'', \mathcal{O}(mK))\\ &\to H^1(M'', \mathcal{O}(mK)) \to \cdots\end{aligned}$$

By Grauert-Riemenschneider vanishing theorem, we have

$$H^1(M', \mathcal{O}(mK)) = 0, \quad \text{and} \quad H^1(M'', \mathcal{O}(mK)) = 0.$$

It follows that

$$H^1_c(M', \mathcal{O}(mK)) = \Gamma(M' - A, \mathcal{O}(mK))/\Gamma(M', \mathcal{O}(mK))$$

and

$$H^1_c(M'', \mathcal{O}(mK)) = \Gamma(M'' - A, \mathcal{O}(mK))/\Gamma(M'', \mathcal{O}(mK)).$$

By Serre duality, we have

$$H^1_c(M', \mathcal{O}(mK)) \cong H^{n-1}(M', \mathcal{O}(K - mK)), \quad H^1_c(M'', \mathcal{O}(mK)) \cong H^{n-1}(M'', \mathcal{O}(K - mK)).$$

It follows from Lemma 2.5 that

$$H^{n-1}(M', \mathcal{O}(K - mK)) \cong H^{n-1}(M'', \mathcal{O}(K - mK)),$$

then

$$H^1_c(M', \mathcal{O}(mK)) \cong H^1_c(M'', \mathcal{O}(mK)).$$

We conclude that

$$\Gamma(M' - A, \mathcal{O}(mK))/\Gamma(M', \mathcal{O}(mK)) \cong \Gamma(M'' - A, \mathcal{O}(mK))/\Gamma(M'', \mathcal{O}(mK)).$$

Thus further by

$$L^{2/m}(M' - A) = \Gamma(M', \mathcal{O}(mK + (m-1)A))$$

and

$$L^{2/m}(M'' - A) = \Gamma(M'', \mathcal{O}(mK + (m-1)A)),$$

we have

$$\Gamma(M' - A, \mathcal{O}(mK))/L^{2/m}(M' - A) \cong \Gamma(M'' - A, \mathcal{O}(mK))/L^{2/m}(M'' - A).$$

It follows that $\delta_m(X)$ is well-defined.

3 Proof of the main theorem

The following proposition was the starting point of our investigation.

Proposition 3.1. *Let X_1 and X_2 be two compact connected, $(2n-1)$-dimensional $(2n-1 \geqslant 3)$, strongly pseudoconvex embeddable CR manifolds which bound complex varieties V_1 and V_2 in $\mathbb{C}^{N_1}$ and $\mathbb{C}^{N_2}$ respectively. Suppose the singular set S_i of V_i, $i = 1, 2$ is either an empty set or a set consisting of only isolated normal singularities. If $\Phi\colon X_1 \to X_2$ is a non-constant CR morphism, then Φ is surjective and Φ can be extended to a proper surjective holomorphic map from V_1 to V_2 such that $\Phi(S_1) \subseteq S_2$, $\Phi^{-1}(X_2) = X_1$ and $\Phi\colon V_1 \setminus \Phi^{-1}(S_2) \to V_2 \setminus S_2$ is a covering map. Moreover, if S_2 does not have quotient singularity, then $\Phi^{-1}(S_2) = S_1$.*

Proof. Let $\phi_1, \ldots, \phi_{N_2}$ be the component functions of Φ. Then ϕ_i as CR holomorphic function on X_1 can be extended in a one sided neighborhood of X_1 in V_1. By Andreotti and Grauert [1, Théorénie 15], ϕ_i can be extended holomorphically to $V_1 - S_1$ where S_1 is the singular set of V_1. Since S_1 is either an empty set or a set consisting of only isolated normal singularities, ϕ_i can be extended holomorphically to V_1.

We claim that $\Phi(V_1) \subseteq V_2$. To see this, let $f_1, \ldots, f_k$ be the defining equations of V_2, i.e., $V_2 = \{y \in \mathbb{C}^{N_2} : f_1(y) = \cdots = f_k(y) = 0\}$. Clearly, $\Phi^*(f_i) = f_i \circ \Phi$ is a holomorphic function on V_1 which vanishes on X_1 for $1 \leqslant i \leqslant k$. Since X_1 is of real codimension one in V_1, $\Phi^*(f_i)$ is identically zero on V_1 for $1 \leqslant i \leqslant k$. This implies that $\Phi(V_1) \subseteq V_2$. By maximum principle, $\Phi(X_1) \cap \Phi(V_1 - X_1) = \phi$. It follows that $\Phi^{-1}(X_2) = X_1$ and Φ is a proper map from V_1 to V_2. By proper mapping theorem, $\Phi(V_1)$ is a complex variety.

We claim that $\dim \Phi(V_1) = n$. If $\dim \Phi(V_1) < n$, then for some q in $\Phi(V_1)$, $\Phi^{-1}(q)$ is a compact variety of dimension at least one sitting inside V_1. This gives a contradiction since V_1 is Stein. As $\Phi(V_1) \subseteq V_2$ and $\dim \Phi(V_1) = n = \dim V_2$, we have $\Phi(V_1) = V_2$. It follows that $\Phi(X_1) = X_2$. A local computation of Fornaess [3, Proposition 12] would apply to show that Φ is a local biholomorphism near X_1. In particular, $\Phi : V_1 - \Phi^{-1}(S_2 \cup \Phi(S_1)) \to V_2 - (S_2 \cup \Phi(S_1))$ is local biholomorphic and hence is a finite covering.

Let $p \in S_1$ and $q = \Phi(p)$. We claim $q \in S_2$. Suppose on the contrary that q is a smooth point in V_2, then Φ maps a neighborhood U_1 of p to a neighborhood U_2 of q as branch covering. Since p is a normal singularity, the punctured neighborhood $U_1 - p$ of p is connected. On the other hand, the punctured neighborhood $U_2 - \{q\}$ of q is simply connected because q is a smooth point. We conclude that $\Phi|_{U_1} : U_1 \to U_2$ is one to one and onto. By Hartog extension theorem, the inverse map $\Phi^{-1}|_{U_2-\{q\}} : U_2 - \{q\} \to U_1 - \{p\}$ can be extended holomorphic to U_2. Hence, $\Phi|_{U_1} : U_1 \to U_2$ is a biholomorphic map. This leads to a contradiction. Therefore, $\Phi(S_1) \subseteq S_2$ and $\Phi : V_1 - \Phi^{-1}(S_2) \to V_2 - S_2$ is a covering map.

Now assume that S_2 does not have quotient singularity. Let q be any point in S_2. We need to show that $\Phi^{-1}(q) \subseteq S_1$. If $\Phi^{-1}(q)$ is not contained in S_1, then there exists a smooth point q' of V_1 in $\Phi^{-1}(q)$. Recall that $\Phi^{-1}(q)$ is a finite set. We can find an open neighborhood U of q' which is biholomorphic to a domain in $\mathbb{C}^n$ such that $\Phi|_U$ from U to the germ of (V_2, q) is a branch covering with ramification locus $\{q'\}$. By [9, Theorem 1], we conclude that (V_2, q) is a quotient singularity. This leads to a contradiction. ☐

Proof of the main theorem. Let V_i be a normal variety in $\mathbb{C}^{N_i}$ with only isolated singularities such that $\partial V_i = X_i, i = 1, 2$. Let S_1 and S_2 be the singular set of V_1 and V_2, respectively. Let $\phi : X_1 \to X_2$ be a non-constant CR morphism. In view of Proposition 3.1, ϕ can be extended to a proper surjective holomorphic map from V_1 and V_2 such that $\phi(S_1) = S_2$, and $\phi : V_1 - \phi^{-1}(S_2) \to V_2 - S_2$ is a covering map. There is a natural map

$$\phi^* : \frac{\Gamma(V_2 - S_2, \mathcal{O}(mK))}{L^{2/m}(V_2 - S_2, \mathcal{O}(mK))} \to \frac{\Gamma(V_1 - \phi^{-1}(S_2), \mathcal{O}(mK))}{L^{2/m}(V_1 - \phi^{-1}(S_2), \mathcal{O}(mK))}.$$

Since $\phi : V_1 - \phi^{-1}(S_2) \to V_2 - S_2$ is a finite covering map, a form $w \in \Gamma(V_2 - S_2, \mathcal{O}(mK))$ is $L^{2/m}$-integrable if and only if $\phi^*(w)$ is $L^{2/m}$-integrable. Thus ϕ^* is injective. Observe that $\phi^{-1}(S_2) - S_1$ is a discrete subset in the smooth part of V_1. By Hartog's theorem, $\Gamma(V_1 - \phi^{-1}(S_2), \mathcal{O}(mK)) = \Gamma(V_1 - S_1, \mathcal{O}(mK))$ and $L^{2/m}(V_1 - \phi^{-1}(S_2), \mathcal{O}(mK)) = L^{2/m}(V_1 - S_1, \mathcal{O}(mK))$. It follows that $\delta_m(X_2) \leqslant \delta_m(X_1)$. ☐

Acknowledgements This work was supported by the Start-Up Fund from Tsinghua University and National Natural Science Foundation of China (Grant No. 11401335).

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